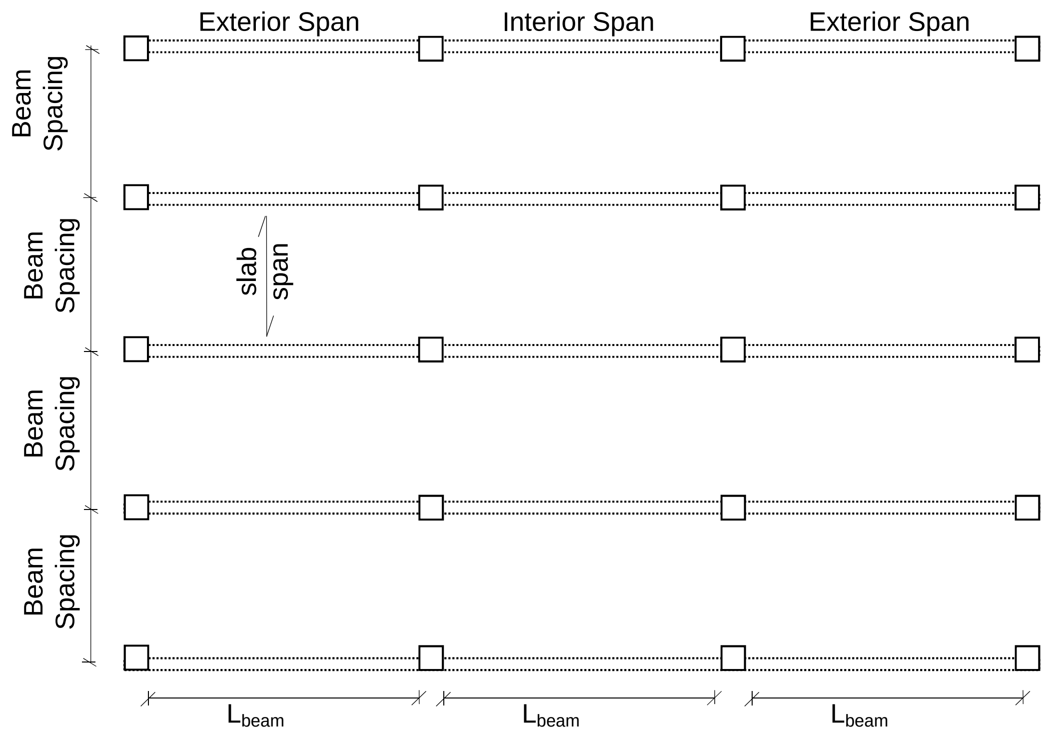
Slab Design Example

Design a one-way slab for an interior bay of a multi-story office building using the information specified below. Neglect compression reinforcement. Assume partitions cannot be damaged by deflections. Round slab thickness up to the nearest ¼".

Use a spreadsheet to design each section and show your results with a sketch (elevation view). Submit hand calculations matching the spreadsheet for one section. Include units on all numbers.



Plan View of Floor System

f'c = 5000 psi	w^{SD} = 10 psf	beam spacing = 18 ft
fy = 60,000 psi	w^{LL} = 100 psf	b_w of beam = 14 in

Slab Thickness, t.

Set h = h_{min} for deflection. Use ACI Table 9.5(a): h_{min} = L / 28,
L = distance from center of support to center of support = beam spacing = 16 ft
h_{min} = (16' x 12"/') / 28 = 6.857 ", use h = 7.0"

Check shear strength: $V_u = 1.15\, w_u\, L_n / 2$ at exterior face of 1st interior support (ACI 8.3, pg 109)

$$w_u = 1.2(w^{self} + w^{SD}) + 1.6w^L$$

$$w^{self} = (0.150^{kcf})(\frac{12"}{12"/'})(\frac{7.75"}{12"/'}) = 0.0969klf$$

$$w_u = 1.2[0.0969^{klf} + (0.010^{ksf})(1')] + 1.6(0.100^{ksf})(1'), \qquad w_u = 0.288klf$$

$$l_n = clear\,span = beam\,spacing - b_{w\,of\,beam} = 16' - \frac{16"}{12"/1}, \qquad l_n = 14.67\,ft$$

$$V_u = 1.15(0.288klf)(14.67\,ft)/2 = 2.79k$$

$$\varphi V_c = (0.75)(2\sqrt{f_c^{'}}\, b_w d)$$

Use #4 bars for flexural reinforcement (revise if needed) — Use #4 bars for flexural reinf.
cover = 0.75 in (ACI 7.7.1, pg 93)

$$d = h - \text{cover} - \frac{\phi_{bar}}{2} = 7.75" - 0.75" - \frac{1}{2}(\frac{4}{8})", \qquad d = 6.75"$$

$$\phi V_c = (0.75)(2\sqrt{4000\,psi}(12in)(6.75in) = 8.59k$$

$$\varphi V_c = 8.59 > 2.79k = V_u,\ OK$$

2. Design Flexure Reinforcement at each Section

Min. Reinforcement:

$$\frac{A_{s\_min}}{bh} = 0.0018 \quad (\text{ACI}\,10.5.4)$$

$$\text{A}_{\text{s\_min}} = (0.0018)(12")(7.75"), \qquad A_{s\_min=} = 0.167\frac{in^2}{ft\ width}$$

Max. Reinforcement Spacing (ACI 10.5.4):

$$S_{\max\_slabs} = \min[18in, 3h] = \min[18in, 3(7.75in)] = 18in$$

Distribution of reinforcement (ACI 10.6.4):

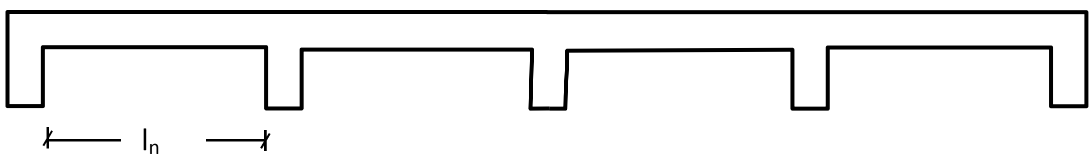
$$S_{max\_distrib} = \min[15\frac{40{,}000}{\frac{2}{3}f_y} - 2.5c_c, 12\frac{40{,}000}{\frac{2}{3}f_y}]$$

$$c_c = h - d - \frac{\varphi_{bar}}{2} = 7.75in - 6.75in - \frac{1}{2}\frac{4}{8} = 0.75in$$

$$S_{max\_distrib} = \min[15\frac{40{,}000}{\frac{2}{3}(60{,}000\,psi)} - 2.5(0.75in), 12\frac{40{,}000}{\frac{2}{3}(60{,}000\,psi)}]$$

$$S_{max} = \min[S_{max\_slabs},\ S_{max\_distrib}] = \min[18in,\ 12in], \qquad S_{max} = 12in$$

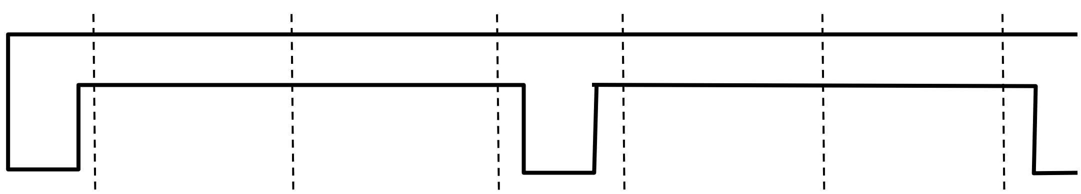
Design of Bar Spacings:



Elevation View of Floor Slab

We can design one half of the slab length due to symmetry.

Design Sections



M Coeff. (ACI 8.3)	1/24	1/14	1/10	1/11	1/16	1/11
M_u, k-ft	3.40	5.83	8.17	7.43	5.10	7.43
$A_{s\_req'd}$, in^2	0.118	0.202	0.283	0.257	0.177	0.257
s, in	12	11.87	8.48	9.33	13.57	9.33
$A_{s\_prov'd}$, in^2	0.20	9	8	8	12	8
a, in	0.235	0.27	0.30	0.30	0.20	0.30
ε_s	0.0659	0.314	0.353	0.353	0.235	0.353
ϕ M_n, k-ft	5.97	0.0486	0.0429	0.0429	0.0659	0.0429

Calculations for Column 2 provided below. Calculations for Columns 3 through 7 performed by spreadsheet (attached)

<u>Typical calcs. for first column of above table:</u>

$$M_u = w_u \frac{l_n^2}{24} = (0.288klf)\frac{(16.83\,ft)^2}{24} = 3.40\,\text{k - ft}$$

$A_{s\_req'd}$:

$$Set\ \varphi M_n = M_u$$

$$\varphi A_s\ f_y(d - \frac{a}{2}) = M_u \quad \text{(Assume steel yields at ultimate strength)}$$

$$\text{Assume: } \text{d-}\frac{\text{a}}{2} \cong 0.95d$$

$$A_{s\_req'd} = \frac{M_u}{\varphi\, f_y\, 0.95d} = \frac{(3.40\text{k-ft})\ (12\frac{in}{ft})}{(0.90)(60ksi)(0.95)(6.75in)} = 0.118\frac{in^2}{ft - width} < A_{s\_\min}$$

$$\therefore A_{s\_req'd} = 0.167\frac{in^2}{ft\ width}$$

Rebar spacing, s:

$$s = \frac{\text{A}_{\text{bar}}}{\text{A}_{\text{s\_req'd}}} = \frac{0.20in^2}{0.167\dfrac{in^2}{12''\ width}} = 14.4in > S_{\max} = 12in$$

$$\therefore s = 12in$$

Strain in Reinforcement, ε_s

$$A_{s\_prov'd} = A_{bar}\ \frac{12}{s} = (0.20\frac{in^2}{bar})(\frac{1\ bar}{12in})(\frac{12in}{ft - width}) = 0.20\frac{in^2}{ft - width}$$

From C = T:

$$0.85 f_c^{'}\, a\, b = A_s\ f_y$$

$$0.85(5ksi)\ a\ (12in) = (0.20in^2)(60ksi), \quad a = 0.235in$$

$$y_t = \frac{a}{\beta_1} = \frac{0.235in}{0.80} = 0.294in$$

$$\frac{0.003}{y_t} = \frac{0.003 + \varepsilon_s}{d}, \quad \frac{0.003}{0.294in} = \frac{0.003 + \varepsilon_s}{6.75in}$$

$$\varepsilon_s = 0.0658$$

$$\therefore \varepsilon_s > 0.004 = \text{ACI min. for beams and slabs, OK}$$

$$\varepsilon_s > 0.005 \text{ so } \ \phi = 0.90 \text{ as assumed, OK}$$

ϕM_n:

$$\phi M_n = \phi\ A_s\ f_y(d - \frac{a}{2}) = (0.90)(0.20in^2)(60ksi)(6.75in - \frac{0.235in}{2})\frac{1ft}{12in}$$

$$\phi M_n = 5.97k - ft$$

<u>Design bar cutoffs:</u>

Extension of top bars past face of support (from Text Figure A-5c) =

$$\frac{l_n}{4} = \frac{176''}{4} = 44'' = 3'8'' \quad \text{at end beams}$$

$$0.3\,l_n = 0.3(176'') = 53'' = 4'5'' \ \text{ at interior beams}$$

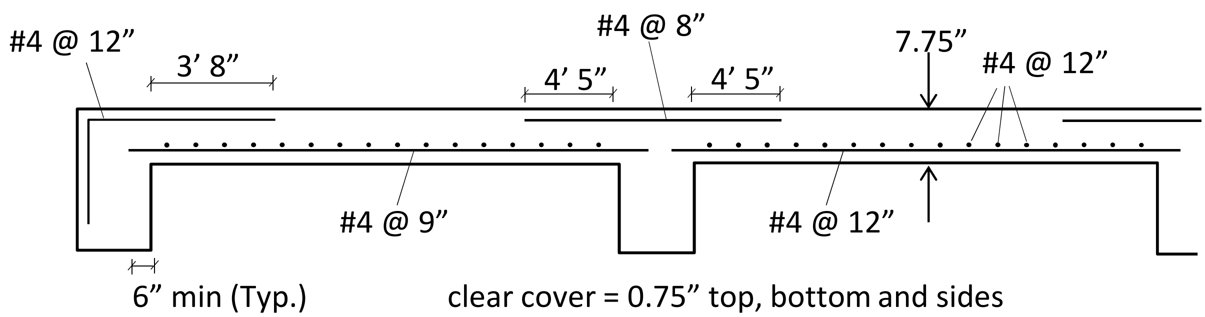
<u>Temperature & Shrinkage Steel:</u>

Use #4 bars (not bent by workmen walking on)

Also, $A_{s\_req'd}$ for T&S reinforcement = $A_{s\_\min}$ for slabs = $0.167\dfrac{in^2}{ft-width}$

$$s = \frac{A_{bar}}{\dfrac{A_{s\_req'd}}{12in}} = \frac{0.20in^2}{\dfrac{0.167in^2}{12in}} = 14.4in,$$

$$s_{\max} = \min[18'', 5h] = \min[18in,\ 5(7.75in)] = 18in \qquad \text{T \& S Reinf : \#4bars @ 12in}$$



<u>Slab Details</u>

Example Slab Design

Material Properties

compressive strength of concrete	f'_c	5,000	psi	
coefficient for depth of stress blocl	β_1	0.80		
yield strength of reinforcement	f_y	60,000	psi	
unit weight of reinf. conc.	UW	150	pcf	
modulus of elasticity of concrete	E_c	4,030,000	psi	= 57000 * SQRT(f'$_c$)
modulus of rupture of concrete	f_r	530	psi	= 7.5 * SQRT(f'$_c$)
modulus of elasticity of steel	E_s	29,000,000	psi	

Geometry

span length of slab (beam spacing	L	18	ft	center to center
width of supporting beam	$b_{support}$	14	in	
clear span	L_n	16.83	ft	= L - $b_{support}$/12
minimum thickness	h_{min}	7.714	in	= L / 28 * 12
depth of slab	h	7.75	in	

Loads

self weight of beam	sw	0.0969	klf	= UW/1000 * h/12 * 1
superimposed dead load	SDL	10	psf	
live load	LL	100	psf	
uniform distributed factored load	ω_u	0.288	klf	

= 1.2 * (sw + SDL/1000 * 1) + 1.6 * LL/1000 * 1

Reinforcement

size of flexural reinforcement	Bar_size	4		
diameter of steel reinf. Bars	ϕ_{bar}	0.5	in	
	A_{bar}	0.20	in^2	
clear cover	cover	0.75	in	
maximum effective depth	d_{max}	6.75	in	=h - cover - ϕ_{bar}/2
effective depth	d	6.75	in	

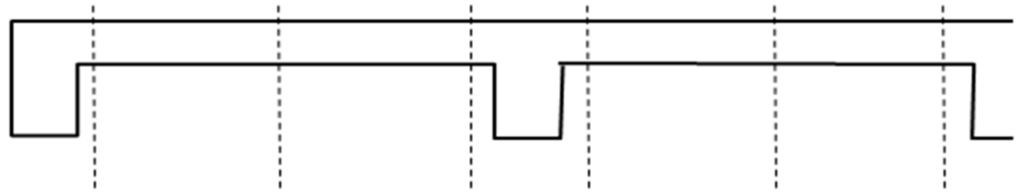
Shear

Shear	OK		= IF(ϕV_n >= V_u, "OK", "NG")	
max shear force	V_u	2.79	k	= 1.15 * ω_u * L_n/2
shear strength of concrete	ϕV_n	8.59	k	= 0.75 * 2 * SQRT(f'$_c$) * 12 * d /1000

Flexure

min. reinforcement	A_{s_min}	0.167	in^2	= 0.0018 * 12 * h
max spacing for slabs	s_{max_slabs}	18.00	in	= MIN(18, 3 * h)
clear distance from reinf. to ten. face	c_c	0.750	in	= h - d - ϕ_{bar}/2
max spacing for distribution	$s_{max\_distrib}$	12.00		
	s_{max}	12.00	in	= MIN(s_{max_slabs}, $s_{max\_distrib}$)

Example Slab Design



coefficient from ACI 8.3	Mcoef	24	14	10	11	16	11	
moment due to factored loads	M_u	3.40	5.83	8.17	7.43	5.10	7.43	k-ft
As_req'd for ϕ Mn to equal Mu	$A_{s\_req'd}$	0.167	0.202	0.283	0.257	0.177	0.257	in^2
s req'd for #4 bar to provide As_req'd	$s_{req'd}$	12.00	11.87	8.48	9.33	13.57	9.33	in
design bar spacing	s	12	9	8	8	12	8	
As for #4 bar at spacing "s"	$A_{s\_prov'd}$	0.20	0.27	0.30	0.30	0.20	0.30	in^2
depth of stress block	a	0.235	0.314	0.353	0.353	0.235	0.353	in
strain in steel reinforcement at ult.	ε_s	0.0659	0.0486	0.0429	0.0429	0.0659	0.0429	
strength reduction factor	ϕ	0.90	0.90	0.90	0.90	0.90	0.90	
available flexure strength	ϕM_n	5.97	7.91	8.87	8.87	5.97	8.87	k-ft

where

M_u	$= \omega_u * L_n$^2 / Mcoef
$A_{s\_req'd}$	= MAX(A_{s_min}, M_u * 12000 / (0.9 * f_y * 0.95 * d))
$s_{req'd}$	= MIN(s_{max}, A_{bar} / ($A_{s\_req'd}$ / 12))
a	= $A_{s\_prov'd}$ * f_y / (0.85 * f'_c * 12)
ε_s	= 0.003 / (a / β_1) * d - 0.003
ϕ	=IF(ε_s < 0.002, 0.65, IF(ε_s > 0.005, 0.9, 0.48 + 83 *ε_s))
ϕM_n	= ϕ * $A_{s\_prov'd}$ * f_y * (d - a/2) /12000